MUSCULAR COORDINATION AND MULTI-DEGREE OF FREEDOM CONTROL SYSTEMS

I. Introduction

In our First Quarterly Progress Report (July 30, 1969) a set of hand-derived equations for the velocity components of the trunk, thigh, shank, foot, upper arm, forearm, and head of the seven element stick man of Figure 1 was presented. Future work was to have been devoted to finding a digital computer method for reliabily carrying out the formation of the expressions for the kinetic and potential energies and the differentiation operations required to obtain the equations of motion using Lagrange's equations.

This work has been completed, and a digital simulation based on these equations is now under development. Preliminary results from this simulation are presented in this report.

After a short summary of the progress to date under this contract, the prior work and concepts that form a context within which the present research is viewed will be presented, and the tasks that must be carried out to provide a satisfactory conclusion to this work will be outlined.

An estimate of the areas in which discovery of control principles useful to NASA seems probable is provided. It is hoped that this section will serve to clarify the relationship of our research to the concerns and goals of NASA/ERC.

A more detailed presentation of the symbolic programming techniques used and the results obtained, followed by an estimate of the direction work will take in the immediate future, completes this chapter.

II. Summary of Progress to Date

1. Derivation of the full nonlinear equations of motion of the seven-element stick man (Figure 1 and Figure 1a) has been completed with the use of a digital symbolic processing language (FORMAC).
2. Small angle approximations have been made leading to a linearized model suitable for small deviations from equilibrium.
3. This small angle model has been simulated digitally, and is producing preliminary results which are being used to verify the validity of the equations of motion. Results from this simulation appear later in this report.
4. Experimentation to date has been with an idealized form of control law, with each joint torqued proportional to the joint angular deflection and proportional to the joint angular rate. It should be noted carefully that we make no assertion that this is the form of control law actually used by human beings; this control law is being used merely for convenience in debugging the small angle simulation.

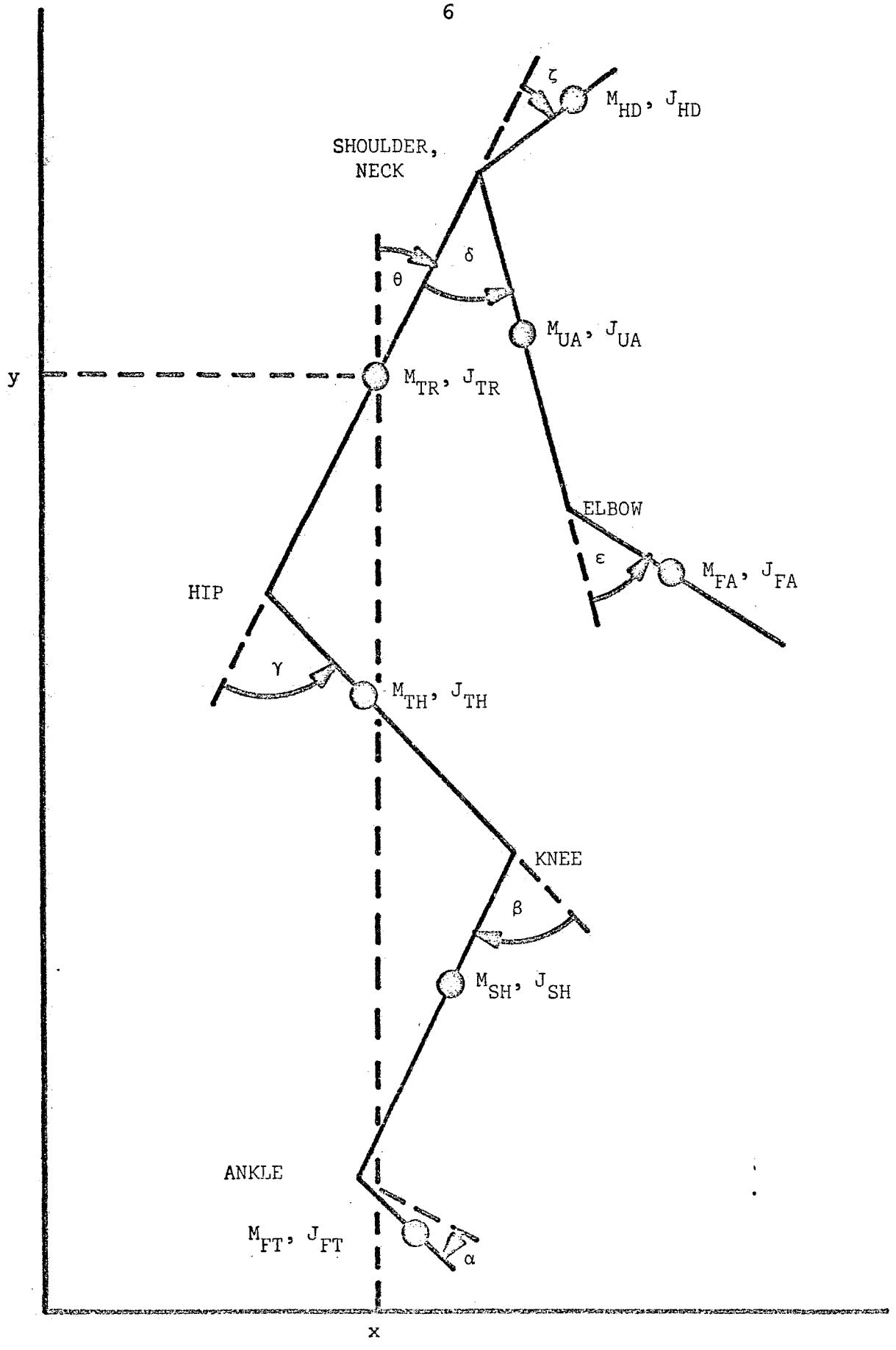


Figure 1. GENERALIZED COORDINATES

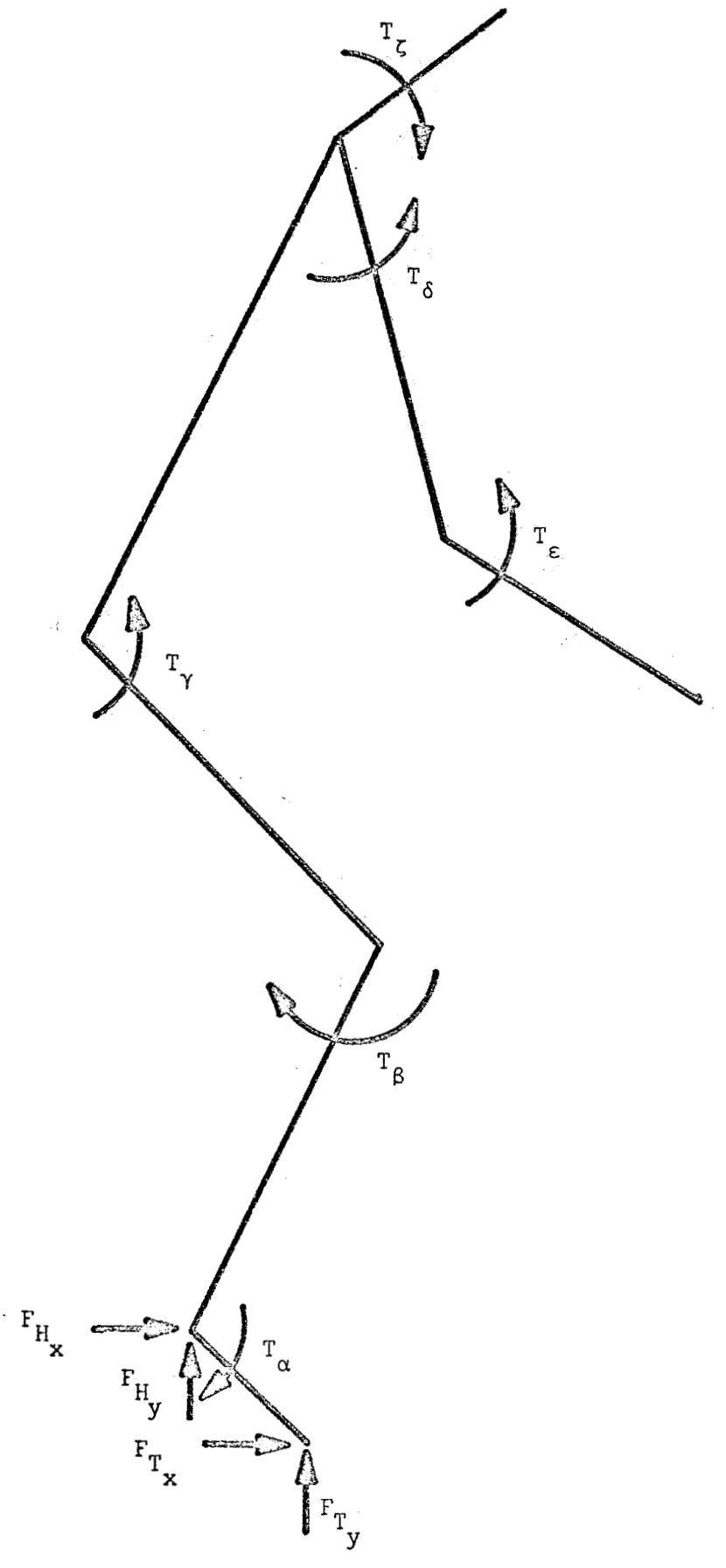


Figure 1a. FORCES AND TORQUES

5. Professor J. C. Hill presented a paper at the 1969 I.E.E.E. Systems Science and Cybernetics Conference (October 22-24, 1969; Philadelphia) in which the analysis and the results presently developed under this contract were presented. A reprint of this paper is appended to this chapter.

III. Background

In view of the complexity of the nonlinear equations of motion that have resulted from this analysis, it is desirable to review the context in which the work is viewed. Accordingly, the background leading to this particular problem will be discussed. Next the hypotheses that differentiate this work from others in the field will be discussed. Finally the broad outline of this research program in Biomechanics will be given.

Multi-Degree of Freedom Systems

First, it should be clearly in mind that our interest is in the control of multi-degree-of-freedom systems, and not in the dynamics of a specific system to be controlled per se. Many examples of multi-degree of freedom systems of considerable technological importance can be cited. Businesses receive or acquire as inputs many types of raw materials, including people, and produce as outputs a wide variety of finished goods. A modern petroleum refinery is a dehumanized version of the same example. A missile guidance and control system receives inputs from a variety of sensors, including perhaps radars, inertial

platforms, and terminal tracking devices, and is supposed to coordinate the motion of control surfaces and/or other attitude controlling devices to steer the missile trajectory through a desired point in space, such as a target or an orbital insertion point. Load and distribution scheduling in a large interconnected electrical power generation and distribution system is a problem of much the same kind.

Therefore, basic control principles and techniques developed for and discovered in one multi-degree of freedom application have a high probability of finding application in others.

It is a characteristic of the times that complex control systems design and analysis techniques, largely developed in and by the aerospace discipline, are being applied with vigor in these other areas. The theme of the 1969 I.E.E.E. Systems Science and Cybernetics Conference was the Modeling and Control of Natural Systems, which various authors construed as meaning sociological systems ("Modeling of Criminal Justice System Operations: An Overview" by R. C. Larson, M.I.T.), natural resource systems ("Herd Management and Modern Control Theory" by R. L. Baer, U. of Pennsylvania), learning systems, economic systems, biological systems, political systems ("Urban Political Simulation" by Whitehed and Smith), pattern recognition, and acaptive systems.

Although some of these "applications" appeared to this attendee to be misdirected, as an indication of the leanings of

a growing segment of the systems science fraternity, the implication is obvious: modern controls research is being directed towards the study of more and more complex systems with less and less well-defined performance indices.

Vertebrate Postural Control

It is generally agreed by workers in the field that the prime example of successful flexible coordination of a multi-degree of freedom system is the vertebrate postural control system, where the term postural control as used by the physiologist refers to all examples of muscular coordination used to control the mechanical position of limbs, trunk, etc. Thus, postural control refers to skeletal muscle systems as opposed to digestive musculature, the diaphragm, and the like.

It is our basic thesis that much can be learned about the control of technological multi-degree of freedom systems by studying the principles that Nature has developed for the vertebrate postural control system. In addition, the vertebrate postural control system is worthy of scientific investigation in its own right.

Overview of Prior Work on Postural Control

A review of the work that has been directed toward the goal of understanding the human postural control system leads to the conclusion that the work may be grouped into five areas (see attached paper by J. C. Hill (Appendix I):

*Analytic Representation of the Force-Velocity Characteristics of Muscle Tissue. [1-8] Aimed toward

characterizing human muscle tissue as an actuator, this work dates back at least to 1935, with Milhorn's account of Houk's work a recent example. This work has been carried out primarily by biologists and physiologists, with engineering a relative newcomer on the scene.

*Physiological Sensor Models. [1, 2, 9-11, 20] Exemplified by the work of Miery and Young on the human vestibular system and the work on the muscle spindle receptor by Houk, Gottlieb, Agarwal, and Stark, this work is aimed at developing transfer functions for the basic sensory elements that serve to instrument the human "air-frame." This area has been developed by and for engineers, and is comparatively recent.

*Human Operator Studies. [12-19] Although physiological sensory mechanisms are involved in this work also, the emphasis is less on modeling the sensory organs than it is on using such models as a guide to the performance of the human operator in tracking and other control tasks. An extensive chain of development by McRuer, Graham, Magdeleno, Shirley and Young, Adams, and Jackson has served to define and develop this area over the past ten years. The output of these studies are typically describing functions explaining the behavior of the human operator as a function of amplitude and frequency; hence a rather abstract view of his behavior is taken based on

experimental data, and the emphasis is normally on emperical description of psychological or perceptual factors rather than on mechanical system properties per se.

*Quadruped Locomotion. [21-29] A distinct niche in this grouping is occupied by the work of McGhee and Tomovic. If an analogy may be drawn to aerospace applications, the work discussed thus far has been concerned with control, whereas McGhee has successfully split off a pure guidance problem from the complex phenomena involved in locomotion. In McGhee's work, the dynamic aspects of the problem are suppressed, and concern is centered on the entirely kinematic question of what configuration should be assumed by the extremeties as a function of time if walking, crawling, running, and other gaits are to result.

*Postural Control. [1, 2, 5] The work of Houk attempts to integrate the data on muscle force-velocity characteristics with data on the muscle spindle receptor to develop equations describing the behavior of an agonist-antagonist muscle pair acting against a pure inertia load. Houk's experimentation was carried out on the human wrist rotation system, and led to the block diagram shown here as Figure 2.

Muscular Coordination

If one wishes to understand human muscular coordination as

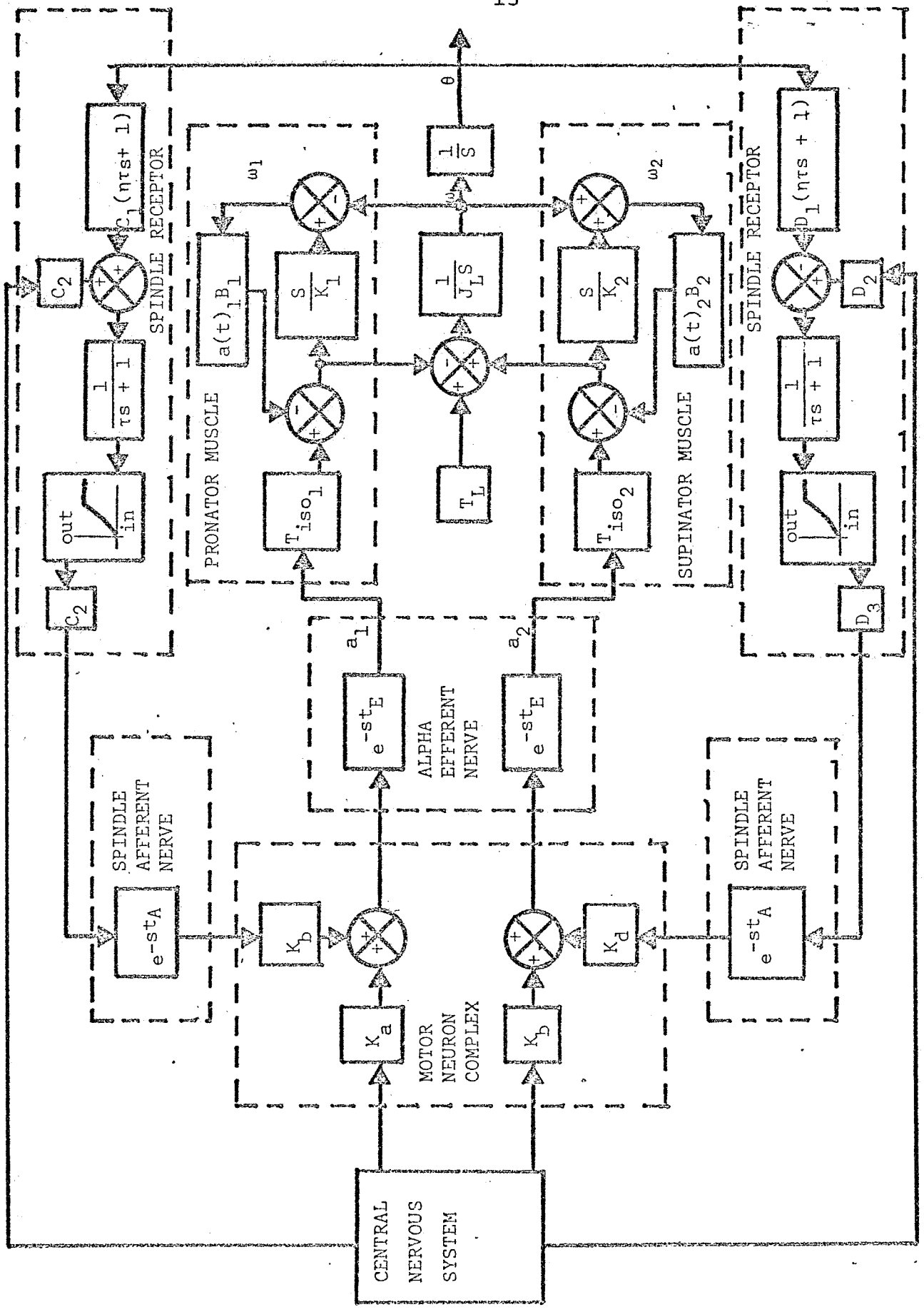


Figure 2. Block diagram of Postural Control System (Wrist Rotation, after Houk)

an example of an exacting multi-degree of freedom control system, it seems improbable that much will be learned about it from a study of Figure 2 and its underlying processes, for the coordination involved in the wrist rotation system is of a very rudimentary sort. The rule might be stated as: Most of the time both the agonist and the antagonist should not both be excited -- don't push and pull at the same time. The coordination involved is analogous to that required of the output transistors in a push-pull audio amplifier. Because the muscles are basically contractile force-producing elements, they must be arranged in pairs in order to produce bipolar torques around a joint. Hence a worthwhile degree of muscular coordination will occur only in a system with more than one joint to be torqued.

The stick man we are studying has six joints to be torqued, the wrist rotation system has only one; is there no middle ground? Yes, there is, and the area is presently being researched by Galiana, [6], who studies the dynamic aspects of legs in biped locomotion. The model has two degrees of freedom, the hip and the knee angles, and is forced by the hip and knee torques. We felt that repetition of Galiana's work would be just that, i.e., repetitive, and therefore chose a more complex model, (which, however, we are attacking from a somewhat different direction with different goals).

Thus Galiana's and our work may be characterized as an attempt to generalize the work of Houk in two directions -- first, increasing the number of joints, and second, extending the load

operated on by the joint musculature away from Houk's pure inertia assumption towards more realistic load dynamics -- in particular, to a manageable segment of the human frame, all in the hope that something can be learned about the principles of muscular coordination from analysis of the resulting models.

Dynamics of the Human Frame

The implicit assumption underlying the desire to study the complex model of the human frame proposed is that if insight is to be gained into Nature's principles of muscular coordination, some appreciation of the control problem she faces must be acquired, i.e., we must study the dynamics of the systems Nature controls so well. It seems unlikely that she would develop a sophisticated control principle that we would find useful in technological multi-degree of freedom systems in order to position a pure inertia load.

Tasks

In carrying out this study, we propose to follow a systems approach to the control of a complex dynamics system, which may be divided into the following three basic steps:

1. Derive the equations of motion, transfer functions, etc., of the basic system to be controlled;
2. Augment this model with a mathematical description of the sensors to be used to measure certain aspects of the state of the system (e.g., feedback pots, tachometers, inertial platforms, etc.)
3. Investigate ways to control the system utilizing the signals provided by the sensors.

Step 1: Basic System Dynamics

It is instructive to examine what one has achieved after completion of each of the above steps. Treating the joint angles and their derivatives as state variables x_i and the joint torques as yet to be specified inputs u_i, the equations of motion would be of the form

$$\dot{\underset{\sim}{x}} = \underset{\sim}{f}(\underset{\sim}{x}, \underset{\sim}{u}) \qquad (1)$$

in standard state variable notation. Given a time history of the muscle torques

$$\underset{\sim}{u} = \underset{\sim}{u}(t) \qquad (2)$$

the motion of the system in response to those torques can be predicted from the solution of the highly nonlinear equation (1). However, the model of equation (1) cannot be experimentally compared with reality, for it is difficult to get a human subject to respond in a controlled torque manner, particularly in nine degrees of freedom. In the human, the muscle torques are not prescribed or controlled as explicit functions of time as in equation (2), but are determined implicitly by the form of unknown control laws, whose nature it is the purpose of our study to determine. The results of step 1 only do not appear directly verifiable experimentally without resort to drugs and/or surgery.

Step 2: Sensor Dynamics

Moving now to step 2, a great deal of information on the sensory mechanisms available to the human control system has been developed by other workers. Houk's work on the stretch reflex [2] and the work of Gottlieb, Agarwal, and Stark [11] on stretch receptor models provide a satisfactory indication of the basic properties of joint angle sensors; the basic conclusion is that (from Gottlieb, Agarwal, and Stark) the form of the rate of afferent nerve firing X is related to the muscle spindle receptor length X_M and force F_M by a transfer relation of the form

$$X = k_1 \frac{s + a}{(s + \frac{1}{T_1})(s + \frac{1}{T_2})} X_M + K_2 \frac{1}{(s + \frac{1}{T_1})(s + \frac{1}{T_2})} F_M \qquad (3)$$

From quick-stretch experimental data on cat muscle the parameters in equation (3) were reported as

$$X = \frac{83.4(s + 3.55)}{(s + 182)(s + 16.7)} X_M + \frac{3200}{(s + 182)(s + 16.7)} F_M \qquad (4)$$

The presence of the zero in the first term of (4) is of interest, for it indicates that at low frequencies the rate of nerve firing is proportional to a linear combination of muscle length and rate of change of muscle length; extrapolating to what would be expected as the net effect of an agonist-antagonist muscle pair, we would expect feedback signals proportional to joint angular rate. The muscle spindle receptor is a tachometer of sorts. We

make the assumption, based on its reasonableness from a control systems point of view, that each joint in the human postural control system is instrumented by a sensor whose equations are of the form of equation (3); the gains and time constants may vary from joint to joint, but the form is invariant. There appears to be no direct physiological data supporting such a gross generalization, nor is there likely to be in the reasonably near future.

Turning now to sensors measuring dynamic variables of a higher order, the work of Miery and Young [9, 10] on the human vestibular system provide basic information on the primarily inertial quantities measured by this apparatus and the transfer functions from sensory input to sensory output. The result is that the semicircular canals measure angular accelerations along three mutually athogonal axes, and that the otolithic organs are vectorial linear acceleration sensors.

Finally, the information provided by the visual sense must be integrated into the control system; what little direct knowledge (from the controls point of view) is known about this subject is reported by Miery, with fringe benefits occurring from scattered statements in the human operator literature.

The parallels between the types of sensors we have discussed here and their aerospace equivalents is striking; the challenge now is to integrate their outputs into a control system capable of mimicking some non-trivial aspect of human behavior.

Assuming now that step 2 has been completed, we are now in possession of a mathematical model of the following form:

$$\dot{\underset{\sim}{x}} = \underset{\sim}{f}(\underset{\sim}{x},\underset{\sim}{u}) \qquad (5)$$

$$\dot{\underset{\sim}{y}} = \underset{\sim}{g}(\underset{\sim}{x},\dot{\underset{\sim}{x}},\underset{\sim}{u}) \qquad (6)$$

Equation (5) describes the plant dynamics; equation (6) describes the sensory dynamics, and we therefore have a completely instrumented "human" at our disposal. The stick man dynamics has provided a framework within which the available sensory dynamic knowledge can be interpreted; the muscles torque the frame, the frame responds, and depending on its motion, the sensors provide outlets.

Step 3: The Control Law

What is now missing is step 3, the real object of our research: a control law that will make the muscle torques, u, dependent on the sensor outputs y and a statement (in some as yet unknown form) of what the system is to do. This last, a generalization of the command input in a conventional control system, appears to be a real conceptual difficulty at the present time. How does one instruct the simulation to stand, lift, walk, jump, sit, or do deep knee-bends without over-structuring its responses? It is proposed to attack this problem gradually, beginning by asking the system to recover its balance from an initially disturbed position, and progressing to more complex tasks as more is learned of the system's behavior from its performance on lower level tasks.

The basic point is that completion of steps 1 and 2 have provided a computer simulation that can predict the consequences of any hypothesized form of control law. Simulation of a proposed law will yield results that are directly comparable with experimental data taken on human subjects performing the same tasks, affording the opportunity to iterate the control law to improve agreement.

As long as the experiments performed may be directly compared with experimental data, our model will be "kept honest" in the most direct way possible: agreement with the system being modeled.

Control Principles

At the present stage of this research, it is impossible to state with any degree of certainty what control principles will result from the studies. However, a few possibilities appear probably enough to warrant further discussion:

*Load Relief: A basic property of the human postural control system is that on occasion it quits trying. If, in attempting to carry out the desired maneuvers, it is in danger of damaging the muscle and/or bone structures involved, sudden relaxation results. This "live to fight another day" strategy is not commonly encountered in technological control systems, whose performance is more usually limited by available power limitations rather than by structural strength limitations. Three exceptions, however, are modern high-performance aircraft, where g-limiting is used; large flexible launch vehicles, where

the control system may function in a bending stress alleviation mode during part of the boost phase; and remote manipulation and exploration machines, where single-minded performance of a specified task by brute force may be disastrous if unexpected obstacles are encountered. All three of these examples are clearly of interest to NASA. How, for example, can one design into a lunar exploration vehicle the ability to decide to roll down (tumbling) a ravine and then get up at the bottom as opposed to fighting desperately to remain upright at the cost of fracturing some critical element? Although rather far away in terms of present knowledge, understanding of this complex extrapolative decision process is a worthwhile eventual goal.

*Anticipation: Cursory examination of the simplest common examples of muscular coordination lead to the conclusion that anticipation is intimately related to load relief. A child jumping off of a chair learns that it must not hit the floor in the completely erect position -- unacceptably large stresses result if it does. Therefore, sometime prior to impact, the frame tends towards an "everywhere bent" configuration that the child has probably learned from experience minimizes something -- the index of performance may be "minimize energy expended subject to the constraint of survival", or some related criterion. At the instant of impact, the legs are already being drawn toward the trunk in an effort to control the peak loads exerted on the feet and toes; i.e., the forthcoming impact is anticipated, not purely responded to. The interesting question is through precisely

what senses this is carried out -- visual, tactile, or inertial, or perhaps a nontrivial combination of the three -- clearly interesting and informative experiments comparing simulation results to real data could be carried out along these lines.

*Sensory Deprivation: In any given experiment, the simulation may easily be deprived of one or more of its sensory channels, and predictions of the effect of a corresponding loss on the human (perhaps an astronaut) could be made. Certain of these results could be easily verified experimentally. At least the visual system may be easily inactivated in a human subject.

*Incipient Tipping: A person, asked to remain erect at a given location, is given a series of harder and harder pushes. For weak pushes he is able to keep his balance without moving his feet, but as the intensity of the pushes increase, a point is eventually reached where he takes one or more hops forward (assuming a feet together constraint is imposed), regains his balance, and hops backwards to the desired spot. If obstacles are present, great versatility in the length and trajectories followed during each hop can result. This is an extremely interesting behavior pattern; it could have great applicability to off-road vehicles, in particular. Again, on what senses does this capability depend? Can it be mimicked to a lower but still technologically useful level?

All of the above-mentioned non-classical control problems may be studied via the approach presently being followed. In

the next section, progress in carrying out the indicated program will be presented.

IV. Progress

As indicated in the First Quarterly Progress Report, expressions for the kinetic and potential energies of the stick man of Figure 1 become so complex that reliable hand calculation of these expressions becomes hopeless. Preliminary study of several computer programs indicated that the FORMAC pre-compiler for PL-1 could carry out the required symbolic manipulations. An IBM 360/65 with this pre-compiler as part of its program library was located at the General Motors Technical Center in Warren, Michigan. This computer was equipped with one million bytes of rapid access core storage, and surprisingly, it turned out that the present problem required it all. Symbolic algebraic processors apparently need fantastic amounts of core.

The FORMAC program we developed (see Appendix II) carried out all the operations needed to derive Lagrange's equations in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial V}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \qquad i = 1,2,\ldots,9 \qquad (7)$$

including:

1. Generation of analytic expressions for T and V.
2. Analytic partial differentiation of these expressions to generate

$$\frac{\partial T}{\partial \dot{q}_i} \text{ , } \frac{\partial T}{\partial q_i} \text{ , and } \frac{\partial V}{\partial q_i} \text{ .}$$

3. Analytic total differentiation of the expression $\frac{\partial T}{\partial \dot{q}_i}$ with respect to time, generating the term

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right).$$

The generalized forces Q_i were derived by hand calculation.

The FORMAC source program (see Appendix II) was over 300 statements long and produced 200 pages of symbolic output; therefore, the output will not be included in this report, but will be provided upon request.

In order to communicate what this program does, a simpler example utilizing the same approach will be presented.

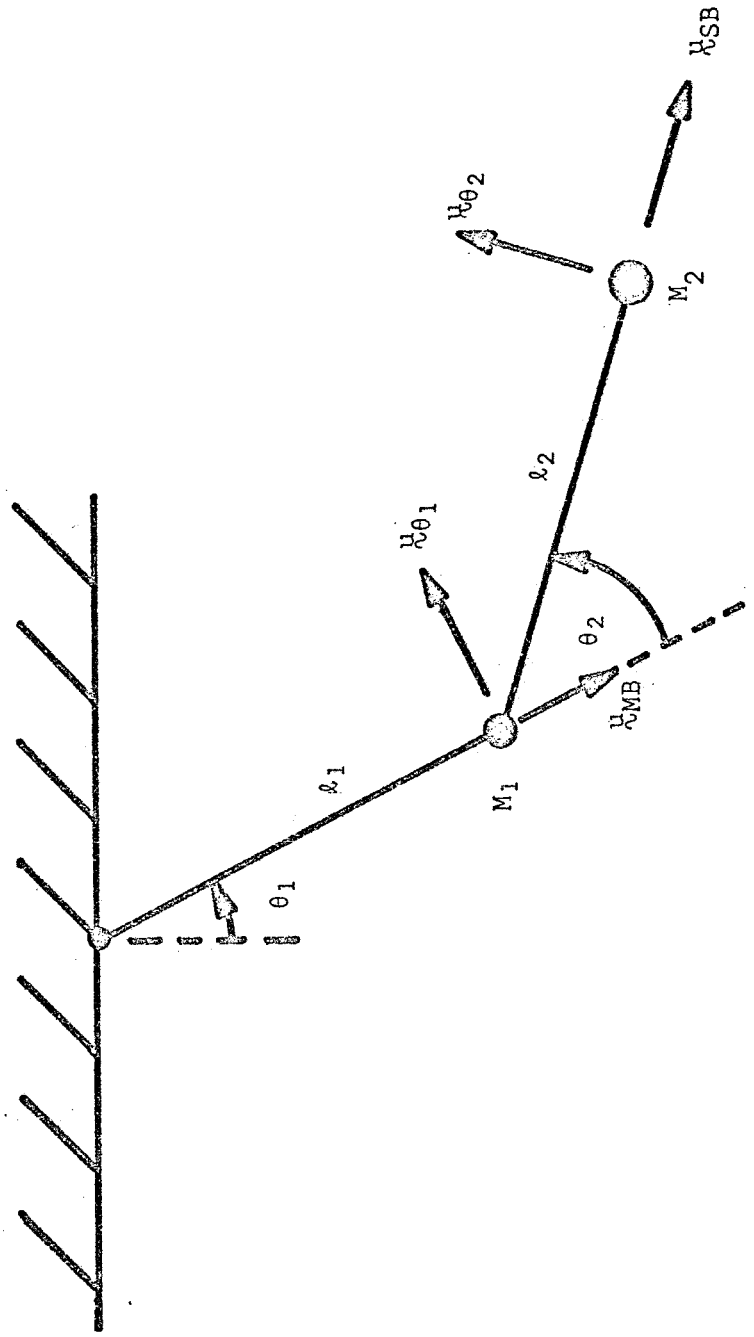
EXAMPLE: We desire the equations of motion of the double Figure 3 in terms of the generalized coordinates θ_1, θ_2 .

For this simple problem, the potential energy may be written as

$$V = (M_1 + M_2)\ell_1 g - (M_1 + M_2)\ell_1 g \cos\theta_1 + M_2\ell_2 g - M_2\ell_2 g \cos(\theta_1 + \theta_2) \tag{8}$$

The velocity of mass 1 is

$$\underset{\sim}{v}_1 = \ell_1\dot{\theta}_1\underset{\sim}{u}_{\theta_1}$$



DOUBLE PENDULUM IN THE GENERALIZED COORDINATES θ_1, θ_2

Figure 3.

where $\underset{\sim}{u}_{\theta_1}$ is a unit vector in the increasing θ_1 direction.

The velocity of mass 2 can be expressed in terms of the velocity of mass 1 by use of the relative velocity theorem as

$$\underset{\sim}{v}_2 = \underset{\sim}{v}_1 + \ell_2\dot{\theta}_2\underset{\sim}{u}_{\theta_2} \tag{10}$$

where $\underset{\sim}{u}_{\theta_2}$ is a unit vector in the increasing θ_2 direction. The addition indicated in (10) is a vector sum; since $\underset{\sim}{v}_1$ as given by (1) and the second term in (10) are not expressed in the same coordinate system, the relation between the coordinate systems must be specified, viz:

$$\underset{\sim}{u}_{\theta_2} = \underset{\sim}{u}_{\theta_1} \cos\theta_2 - \underset{\sim}{u}_{MB} \sin\theta_2 \tag{11}$$

$$\underset{\sim}{u}_{SB} = \underset{\sim}{u}_{\theta_1} \sin\theta_2 + \underset{\sim}{u}_{MB} \cos\theta_2 \tag{12}$$

where $\underset{\sim}{u}_{MB}$ and $\underset{\sim}{U}_{SB}$ are unit vectors directed as shown in Figure 3.

The FORMAC program of Figure 4

1. Sets up V
2. Calculates $\underset{\sim}{v}_1$ (VELM1)
3. Enters the relation between the coordinate systems*
4. Evanuates $\underset{\sim}{v}_2$ (VELM2)
5. Computes $\frac{\partial V}{\partial \theta_1}$ and $\frac{\partial V}{\partial \theta_2}$ (DVT1 and DVT2)

---

*Note that besides working with symbols, FORMAC is also manipulating vector expressions, neither of which a computer is commonly considered able to do.

6. Evaluates T (KE)

7. Evaluates $\frac{\partial T}{\partial \theta_1}$ and $\frac{\partial T}{\partial \theta_2}$ (DTT1 and DTT2)

8. Evaluates $\frac{\partial T}{\partial \dot{\theta}_1}$ and $\frac{\partial T}{\partial \dot{\theta}_2}$ (DTDT1 and DTDT2)

9. Specifies θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$ as functions of time

10. Evaluates $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right)$ and $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right)$ (DTDT1T and DTDT2T)

11. Adds $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)$, $\frac{\partial T}{\partial \dot{q}_i}$, and $\frac{\partial V}{\partial q_i}$ to get Q_i.

Output from this program is presented on next two pages. All intermediate results have been dumped, and the final answers appear as the last nine lines of symbolic output. We have

```
Q1 = SIN ( THETA1 ) G L1 M1 + DTHTA1(1) . ( T ) L1**2 M1 + SIN
-------------------------------------------------------------
( THETA1 ) G L1 M2 - THETA2(1) . ( T ) SIN ( THETA2. ( T )) DTHTA2.
-------------------------------------------------------------
( T ) L2 L1 M2 + DTHTA2(1). ( T ) COS ( THETA2. ( T) ) L2 L1 M2
-------------------------------------------------------------
+ SIN ( THETA1 + THETA2 ) L2 G M2 + DTHTA1(1). ( T ) L1**2 M2
-------------------------------------------------------------

Q2 = SIN ( THETA2 ) DTHTA1 L2 L1 M2 DTHTA2 - THETA2(1) . ( T )
-------------------------------------------------------------
SIN ( THETA2. ( T ) ) DTHTA1. (T ) L2 L1 M2 + DTHTA1(1) . ( T )
-------------------------------------------------------------
COS ( THETA2. (T ) ) L2 L1 M2 + SIN (THETA1  + THETA2 ) L2 G M2
-------------------------------------------------------------
+ DTHTA2(1). ( T ) SIN**2 ( THETA2. ( T ) ) L2**2 M2 + DTHTA2(1) .
-------------------------------------------------------------
( T ) COS**2 ( THETA2. ( T ) ) L2**2 M2
---------------------------------------
```

```
INPUT TO FORMAC PREPROCESSOR
PENDU: PROCEDURE OPTIONS(MAIN);
FORMAC_OPTIONS;
OPTSET(LINELENGTH=72); OPTSET(PRINT); OPTSET(EXPND);
/*                                                              */
     /*  ENTER POTENTIAL ENERGY                                 */
/*                                                              */
LET(V=(M1+M2)*L1*G-(M1+M2)*L1*G*COS(THETA1));
LET(V=V+M2*E2*G-M2*L2*G*COS(THETA1+THETA2));
/*                                                              */
     /*  ENTER VELOCITY OF MASS 1                               */
/*                                                              */
LET(VELM1=DTHTA1     *L1*UTHTA1);
LET(UTHTA2=UTHTA1*COS(THETA2     )-UMB*SIN(THETA2     ));
LET(USB=UTHTA1*SIN(THETA2     )+UMB*COS(THETA2     ));
/*                                                              */
     /*  FIND VELOCITY OF MASS 2 USING RELATIVE VEL. THEORM     */
/*                                                              */
LET(VELM2=DTHTA2     *L2*UTHTA2+VELM1);
/*                                                              */
     /*  DV/QI                                                  */
/*                                                              */
LET(DVT1=DERIV(V,THETA1     ));
LET(DVT2=DERIV(V,THETA2     ));
/*                                                              */
     /*  KINETIC ENERGY                                         */
/*                                                              */
LET(KE=1/2*M1*(COEFF(VELM1,UTHTA1)**2+COEFF(VELM1,UMB)**2));
LET(KE=KE+1/2*M2*(COEFF(VELM2,UTHTA1)**2+COEFF(VELM2,UMB)**2));
/*                                                              */
     /*  DT/QI                                                  */
/*                                                              */
LET(DTT1=DERIV(KE,THETA1     ));
LET(DTT2=DERIV(KE,THETA2     ));
/*                                                              */
     /*  DT/QIDOT                                               */
/*                                                              */
LET(DTDT1     =DERIV(KE,DTHTA1     ));
LET(DTDT2     =DERIV(KE,DTHTA2     ));
     LET(DTDT1=REPLACE(DTDT1,DTHTA1,DTHTA1.(T),DTHTA2,DTHTA2.(T)));
     LET(DTDT1=REPLACE(DTDT1,THETA1,THETA1.(T),THETA2,THETA2.(T)));
     LET(DTDT2=REPLACE(DTDT2,DTHTA1,DTHTA1.(T),DTHTA2,DTHTA2.(T)));
     LET(DTDT2=REPLACE(DTDT2,THETA1,THETA1.(T),THETA2,THETA2.(T)));
/*                                                              */
     /*  D/DT OF DT/QIDOT                                       */
/*                                                              */
LET(DTDT1T=DERIV(DTDT1     ,T));
LET(DTDT2T=DERIV(DTDT2     ,T));
/*                                                              */
     /*  EQUATIONS OF MOTIONS                                   */
/*                                                              */
LET(Q1=DVT1-DTT1+DTDT1T);
LET(Q2=DVT2-DTT2+DTDT2T);
END PENDU;
```

Figure 4. FORMAC Program to Generate Equations of Motion of Double Pendulum.

leading to the equations of motion

$$M_1\ell_1^2 + M_2\ell_1^2\ddot{\theta}_1 + \ell_1\ell_2 M_2 \cos\theta_2\ddot{\theta}_2 - \sin\theta_2\dot{\theta}_2^2 + M_1 + M_2)\ell_1 g \sin\theta_1 + M_2\ell_2 g \sin(\theta_1 + \theta_2) = Q_1 \quad (13)$$

$$M_2\ell_2^2\ddot{\theta}_2 + M_2\ell_1\ell_2 \cos\theta_2\ddot{\theta}_1 - M_2\ell_1\ell_2 \left[\dot{\theta}_1\dot{\theta}_2 \sin\theta_2 - \dot{\theta}_1\dot{\theta}_2\right] \sin\theta_2 + M_2\ell_2 g \sin(\theta_1 + \theta_2) = Q_2 \quad (14)$$

Following this procedure on the seven element stick man leads to a complex set of equations. We have calculated these equations completely, but they do not appear amenable to straight forward machine solution at the present time (basically because they are prohibitively long.)

However, if small angle approximations are made and only terms in the first power in each state variable are kept, the resulting linearized equations may be put in the form

$$\underset{\sim}{A}\ddot{\underset{\sim}{x}} = \underset{\sim}{B}\dot{\underset{\sim}{x}} + \underset{\sim}{C}\underset{\sim}{x} + \underset{\sim}{D}\underset{\sim}{f} + \underset{\sim}{E}\underset{\sim}{T} + \underset{\sim}{V} \quad (15)$$

where

```
V = G L1 M1 - COS ( THETA1 ) G L1 M1 + G L1 M2 - COS ( THETA1 ) G L1 M2
V = G L1 M1 - COS ( THETA1 ) G L1 M1 + G L1 M2 - COS ( THETA1 ) G L1 M2
 + L2 G M2 - COS ( THETA1 + THETA2 ) L2 G M2
VELM1 = DTHTA1 L1 UTHTA1
UTHTA2 = COS ( THETA2 ) UTHTA1 - SIN ( THETA2 ) UMB
USB = SIN ( THETA2 ) UTHTA1 + COS ( THETA2 ) UMB
VELM2 = COS ( THETA2 ) L2 DTHTA2 UTHTA1 + DTHTA1 L1 UTHTA1 - SIN (
THETA2 ) L2 DTHTA2 UMB
DVT1 = SIN ( THETA1 ) G L1 M1 + SIN ( THETA1 ) G L1 M2 + SIN ( THETA1 +
THETA2 ) L2 G M2
DVT2 = SIN ( THETA1 + THETA2 ) L2 G M2
                  2   2
KE = 1/2 DTHTA1   L1   M1
                                                              2   2
KE = COS ( THETA2 ) DTHTA1 L2 L1 M2 DTHTA2 + 1/2 DTHTA1   L1   M1 + 1/2
      2   2                2                2               2          2
DTHTA1   L1   M2 + 1/2 SIN  ( THETA2 ) L2   M2 DTHTA2   + 1/2 COS  (
            2              2
THETA2 ) L2   M2 DTHTA2
DTT1 = 0
DTT2 =  - SIN ( THETA2 ) DTHTA1 L2 L1 M2 DTHTA2
                                                    2                2
DTDT1 = COS ( THETA2 ) L2 L1 M2 DTHTA2 + DTHTA1 L1   M1 + DTHTA1 L1   M2
           2                2                2               2
DTDT2 = SIN  ( THETA2 ) L2   M2 DTHTA2 + COS  ( THETA2 ) L2   M2 DTHTA2
 + COS ( THETA2 ) DTHTA1 L2 L1 M2
                          2
DTDT1 = DTHTA1.( T ) L1   M1 + COS ( THETA2 ) DTHTA2.( T ) L2 L1 M2 +
                  2
DTHTA1.( T ) L1   M2
                          2
DTDT1 = DTHTA1.( T ) L1   M1 + COS ( THETA2.( T ) ) DTHTA2.( T ) L2 L1
                        2
M2 + DTHTA1.( T ) L1   M2
                                                2
DTDT2 = COS ( THETA2 ) DTHTA1.( T ) L2 L1 M2 + SIN  ( THETA2 ) DTHTA2.(
```

```
                                                                                
                                                                                
      2          2                                  2
T ) L2   M2 + COS  ( THETA2 ) DTHTA2.( T ) L2   M2
-------------------------------------------------

                                                                  2
DTDT2 = COS ( THETA2.( T ) ) DTHTA1.( T ) L2 L1 M2 + SIN  ( THETA2.( T )
-------------------------------------------------------------------------

                  2          2                                  2
 ) DTHTA2.( T ) L2   M2 + COS  ( THETA2.( T ) ) DTHTA2.( T ) L2   M2
-------------------------------------------------------------------

                  (1)          2                  (1)
DTDT1T = DTHTA1    .( T ) L1   M1 - THETA2    .( T ) SIN ( THETA2.( T )
-------------------------------------------------------------------------

                                    (1)
 ) DTHTA2.( T ) L2 L1 M2 + DTHTA2    .( T ) COS ( THETA2.( T ) ) L2 L1
-------------------------------------------------------------------------

          (1)          2
M2 + DTHTA1    .( T ) L1   M2
-----------------------------

                    (1)
DTDT2T =  - THETA2    .( T ) SIN ( THETA2.( T ) ) DTHTA1.( T ) L2 L1 M2
-------------------------------------------------------------------------

          (1)                                                  (1)
 + DTHTA1    .( T ) COS ( THETA2.( T ) ) L2 L1 M2 + DTHTA2    .( T ) SIN
-------------------------------------------------------------------------

2                          2                  (1)          2                2
  ( THETA2.( T ) ) L2   M2 + DTHTA2    .( T ) COS  ( THETA2.( T ) ) L2
-------------------------------------------------------------------------

 M2
---

                                          (1)          2
Q1 = SIN ( THETA1 ) G L1 M1 + DTHTA1    .( T ) L1   M1 + SIN ( THETA1 )
-------------------------------------------------------------------------

                  (1)
G L1 M2 - THETA2    .( T ) SIN ( THETA2.( T ) ) DTHTA2.( T ) L2 L1 M2 +
-------------------------------------------------------------------------

      (1)
DTHTA2    .( T ) COS ( THETA2.( T ) ) L2 L1 M2 + SIN ( THETA1 + THETA2 )
-------------------------------------------------------------------------

          (1)          2
 L2 G M2 + DTHTA1    .( T ) L1   M2
-----------------------------------

                                                  (1)
Q2 = SIN ( THETA2 ) DTHTA1 L2 L1 M2 DTHTA2 - THETA2    .( T ) SIN (
-------------------------------------------------------------------------

                                        (1)
THETA2.( T ) ) DTHTA1.( T ) L2 L1 M2 + DTHTA1    .( T ) COS ( THETA2.( T
-------------------------------------------------------------------------

                                                  (1)          2
 ) ) L2 L1 M2 + SIN ( THETA1 + THETA2 ) L2 G M2 + DTHTA2    .( T ) SIN
-------------------------------------------------------------------------

                  2          (1)          2                2
( THETA2.( T ) ) L2   M2 + DTHTA2    .( T ) COS  ( THETA2.( T ) ) L2
-------------------------------------------------------------------------

M2
--
```

$$\underset{\sim}{x} = \begin{Bmatrix} x \\ y \\ \theta \\ \alpha \\ \beta \\ \gamma \\ \delta \\ \varepsilon \\ \zeta \end{Bmatrix}, \quad \dot{\underset{\sim}{x}} = \begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \\ \dot{\delta} \\ \dot{\varepsilon} \\ \dot{\zeta} \end{Bmatrix}, \quad \ddot{\underset{\sim}{x}} = \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ \ddot{\alpha} \\ \ddot{\beta} \\ \ddot{\gamma} \\ \ddot{\delta} \\ \ddot{\varepsilon} \\ \ddot{\zeta} \end{Bmatrix}, \quad \underset{\sim}{f} = \begin{Bmatrix} F_{Hx} \\ F_{Tx} \\ F_{Hy} \\ F_{Ty} \\ F_{Tx}\theta \\ F_{Hy}\theta \\ F_{Ty}\theta \\ F_{Tx}\gamma \\ F_{Hy}\gamma \\ F_{Ty}\gamma \\ F_{Tx}\beta \\ F_{Hy}\beta \\ F_{Ty}\beta \\ F_{Tx}\alpha \end{Bmatrix}, \quad \underset{\sim}{T} = \begin{Bmatrix} T_\gamma \\ T_\beta \\ T_\alpha \\ T_\delta \\ T_\varepsilon \\ T_\zeta \end{Bmatrix}, \quad \underset{\sim}{V} = \begin{Bmatrix} V_x \\ V_y \\ V_\theta \\ V_\alpha \\ V_\beta \\ V_\gamma \\ V_\delta \\ V_\varepsilon \\ V_\zeta \end{Bmatrix} \tag{16}$$

and A, B, C, D, and E are matrices of appropriate size to be conformable with the indicated vectors. In equation (16), T_γ, T_β, etc., are the torques about the joints whose angles are γ, β, etc.; F_{Hx}, f_{Hy} are the x, y components of the ground reaction force on the heel, and F_{Tx}, F_{TY} are the x, y components of the ground reaction force on the toe (see Figure Ia).

The A matrix is given in equation (17).

$$A = \begin{bmatrix} 4.800 & 0.000 & -1.879 & 1.589 & -0.569 & 0.000 & 0.079 & 0.039 & 0.239 \\ 0.000 & 4.800 & 0.000 & 0.000 & 0.000 & -0.019 & 0.000 & 0.000 & 0.000 \\ 0.839 & 0.000 & 4.141 & -3.041 & 0.763 & 0.005 & 0.030 & 0.023 & 0.204 \\ 1.589 & 0.000 & -2.117 & 2.117 & -0.820 & -0.005 & 0.000 & 0.000 & 0.000 \\ -0.569 & 0.000 & 0.763 & -0.763 & 0.763 & 0.005 & 0.000 & 0.000 & 0.000 \\ 0.000 & -0.019 & 0.005 & -0.005 & 0.005 & 0.008 & 0.000 & 0.000 & 0.000 \\ 0.180 & 0.000 & 0.030 & 0.000 & 0.000 & 0.000 & 0.065 & 0.048 & 0.000 \\ 0.039 & 0.000 & -0.009 & 0.000 & 0.000 & 0.000 & 0.009 & 0.024 & 0.000 \\ 0.239 & 0.000 & 0.203 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.203 \end{bmatrix} \quad (17)$$

Since A is <u>not</u> sparse, nearly every acceleration couples into nearly every other one, and we have a bad algebraic loop problem. Analog simulation is indicated, but every nonzero coefficient in A requires an analog pot, as is also true for B, C, D, and E; the number of pots required is more than we have at our disposal.

However, premultiplication of (15) by A^{-1} yields

$$\ddot{\underset{\sim}{x}} = (A^{-1}C)\underset{\sim}{x} + (A^{-1}D)\underset{\sim}{f} + (A^{-1}E)\underset{\sim}{T} + (A^{-1}V) \qquad (18)$$

and in this form there is no algebriac loop problem.

It turns out that $C \equiv 0$; the rest of the matrices in (18) are as follows:

$$A^{-1}C_{\tilde{x}} = \begin{bmatrix} 0.000 & 0.000 & -8.343 & 2.253 & -0.730 & 0.000 & 3.295 & -0.170 & 1.023 \\ 0.000 & 0.000 & -0.041 & 0.052 & 0.057 & 0.000 & -0.005 & 0.000 & -0.001 \\ 0.000 & 0.000 & 41.914 & -85.201 & -31.860 & 0.000 & 1.160 & -0.917 & 5.503 \\ 0.000 & 0.000 & 117.037 & -155.735 & -60.671 & 0.000 & -1.322 & -0.788 & 4.732 \\ 0.000 & 0.000 & 92.987 & -92.958 & -53.470 & 0.000 & -0.015 & 0.000 & -0.004 \\ 0.000 & 0.000 & -10.017 & 12.574 & 13.827 & 0.000 & -1.383 & 0.071 & -0.429 \\ 0.000 & 0.000 & -116.980 & 79.197 & 32.921 & 0.000 & -154.551 & 1.261 & -7.571 \\ 0.000 & 0.000 & 70.551 & -62.789 & -22.151 & 0.000 & 50.783 & -0.511 & 3.069 \\ 0.000 & 0.000 & -32.099 & 82.549 & 32.720 & 0.000 & -5.037 & 1.117 & -6.706 \end{bmatrix} \begin{Bmatrix} x \\ y \\ \theta \\ \gamma \\ \beta \\ \alpha \\ \delta \\ \varepsilon \\ \xi \end{Bmatrix} \quad (19)$$

$$A^{-1}Df = \begin{bmatrix} -0.267 & -0.267 & -0.000 & -0.041 & -0.146 & -0.589 & 0.691 & 0.146 & 0.431 & -0.310 & -0.146 & -0.162 & 0.234 & -0.146 \\ 0.005 & 0.005 & 0.210 & -0.069 & 0.283 & -0.003 & -0.005 & -0.283 & 0.003 & 0.005 & 0.283 & -0.004 & -0.005 & 0.283 \\ 0.381 & 0.381 & 0.000 & 0.092 & 0.325 & -6.718 & 4.240 & -0.325 & 5.863 & 1.876 & 0.325 & -2.658 & -0.521 & 0.325 \\ 1.499 & 1.499 & 0.000 & 0.108 & 0.382 & -7.189 & 5.116 & -0.382 & 6.454 & 3.025 & 0.382 & -2.448 & -0.611 & 0.382 \\ -1.053 & -1.053 & -0.003 & -0.364 & -1.283 & 1.057 & 3.364 & 1.283 & -1.056 & -1.053 & -1.283 & 2.057 & 2.056 & -1.283 \\ 1.217 & 1.217 & 0.469 & -66.609 & 68.041 & -0.857 & -1.395 & -68.041 & 0.923 & 1.235 & 68.041 & -1.036 & -1.203 & 68.041 \\ 0.186 & 0.186 & -0.000 & -0.003 & -0.013 & 7.857 & -5.630 & 0.013 & -6.682 & -1.196 & -0.013 & 2.960 & 0.021 & -0.013 \\ 0.498 & 0.498 & 0.000 & 0.101 & 0.356 & -4.303 & 2.446 & -0.356 & 3.826 & 1.602 & 0.356 & -1.762 & -0.570 & 0.356 \\ -0.066 & -0.066 & -0.000 & -0.043 & -0.153 & 7.412 & -5.053 & 0.153 & -6.371 & -1.510 & -0.153 & 2.849 & 0.245 & -0.153 \end{bmatrix} \begin{Bmatrix} F_{Hx} \\ F_{Tx} \\ F_{Hy} \\ F_{Ty} \\ F_{Tx\theta} \\ F_{Hy\theta} \\ F_{Ty\theta} \\ F_{Tx\gamma} \\ F_{Hy\gamma} \\ F_{Ty\gamma} \\ F_{Tx\beta} \\ F_{Hy\beta} \\ F_{Ty\beta} \\ F_{Tx\alpha} \end{Bmatrix} \quad (20)$$

$$A^{-1}_{ET} = \begin{bmatrix} 0.2065 & 0.3046 & 0.2169 & -0.3327 & -0.1247 & -0.3694 \\ -0.0003 & -0.0036 & 0.4655 & 0.0005 & 0.0002 & 0.0006 \\ 2.4660 & 3.1600 & 3.3551 & 0.5482 & -0.1575 & 0.0407 \\ 3.0814 & 4.5302 & 4.7590 & 0.7989 & -0.0635 & 0.3191 \\ 0.7700 & 2.9120 & 2.1434 & 0.0015 & 0.0005 & 0.0017 \\ -0.0867 & -0.8645 & 111.7349 & 0.1397 & 0.0523 & 0.1551 \\ -0.8632 & -3.7530 & -3.7609 & 20.8373 & -19.4339 & 0.7360 \\ 1.5881 & 2.0013 & 2.2146 & -6.7716 & 47.1390 & 0.3408 \\ -2.7090 & -3.5184 & -3.6103 & -0.1567 & 0.3042 & 5.2958 \end{bmatrix} \begin{Bmatrix} T_\gamma \\ T_\beta \\ T_\alpha \\ T_\delta \\ T_\varepsilon \\ T_\xi \end{Bmatrix} \quad (21)$$

$$A^{-1}V = \begin{Bmatrix} -3.0199 \\ -33.5509 \\ 0.9946 \\ 3.3083 \\ 0.9180 \\ -2.2267 \\ 57.5667 \\ -67.0540 \\ 40.4406 \end{Bmatrix} \quad (22)$$

For convenience in debugging the simulation, a simple control law relating the joint torques to the joint angles and angular rates of the following form was used:

$$
\begin{aligned}
T_{\alpha} &= K_{\alpha}(\alpha) + K_{\dot{\alpha}}(\dot{\alpha}) \\
T_{\beta} &= K_{\beta}(\beta) + K_{\dot{\beta}}(\dot{\beta}) \\
T_{\gamma} &= K_{\gamma}(\gamma) + K_{\dot{\gamma}}(\dot{\gamma}) \\
T_{\delta} &= K_{\delta}(\delta) + K_{\dot{\delta}}(\dot{\delta}) \\
T_{\varepsilon} &= K_{\varepsilon}(\varepsilon) + K_{\dot{\varepsilon}}(\dot{\varepsilon}) \\
T_{\zeta} &= K_{\zeta}(\zeta) + K_{\dot{\zeta}}(\dot{\zeta})
\end{aligned}
\qquad (23)
$$

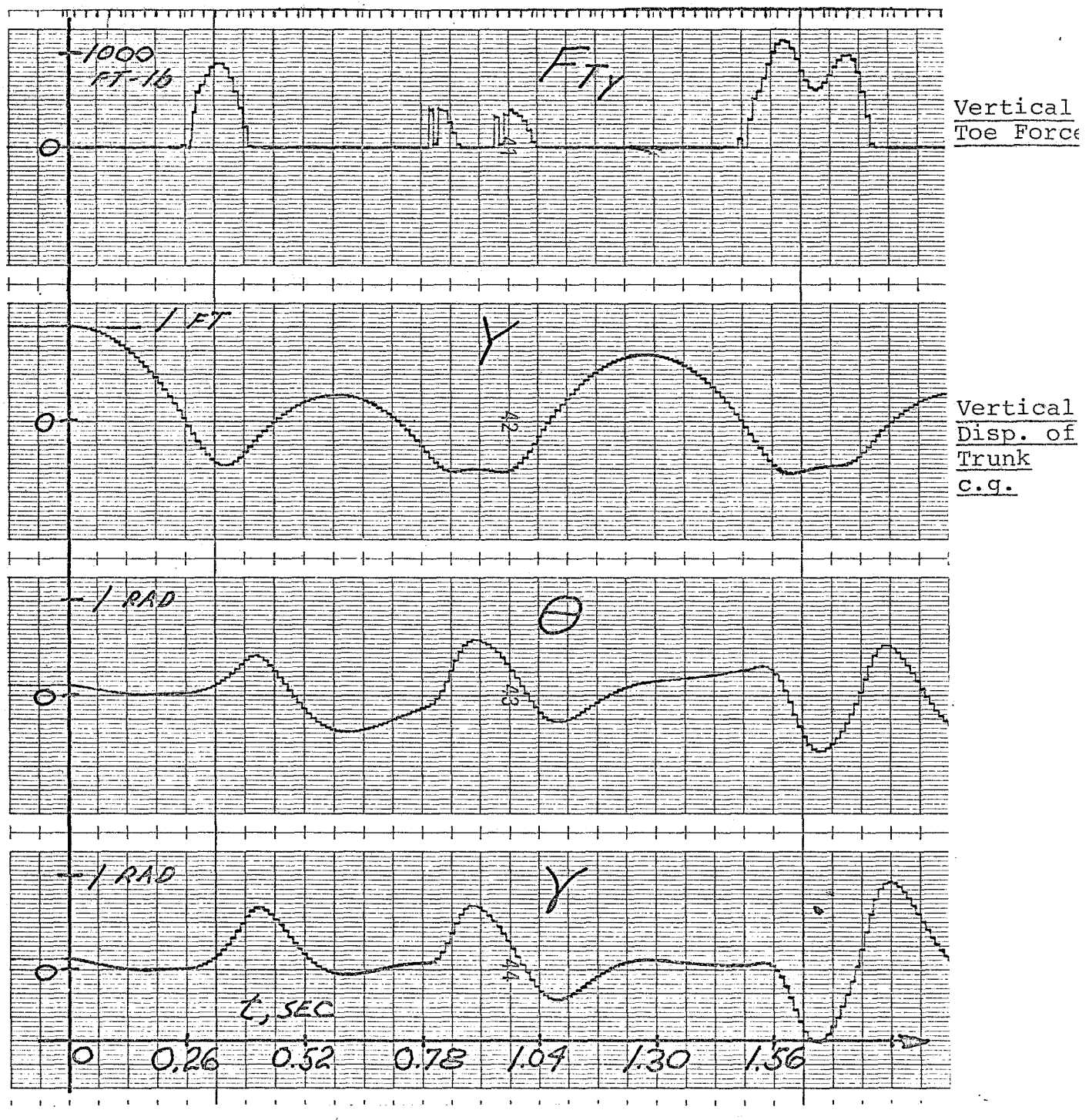


Figure 5. Joint Angles for a One Foot Drop.

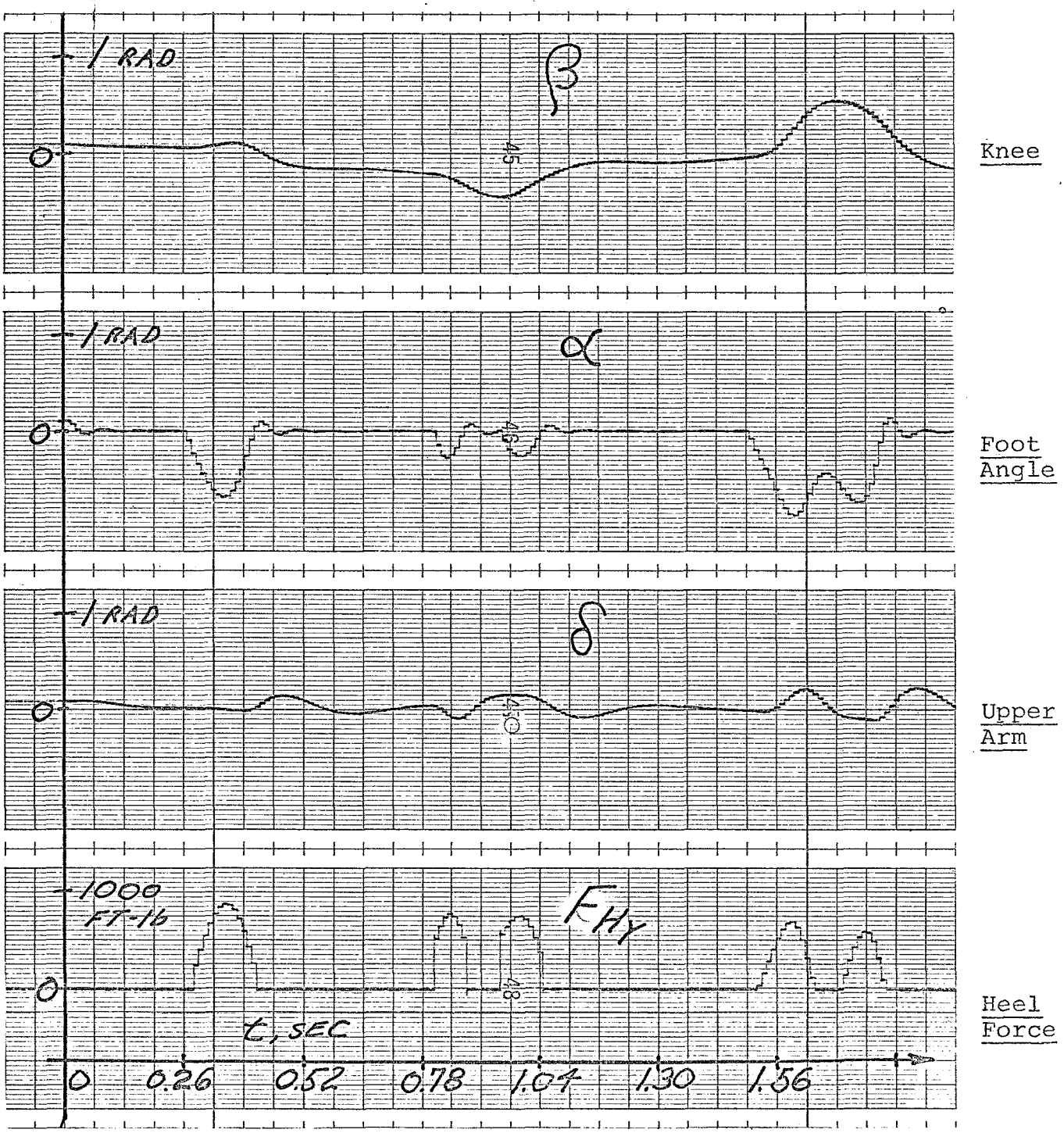


Figure 5 (Cont'd). Joint Angles for a One Foot Drop.

i.e., each joint was torqued proportional to a linear combination of joint angle and joint angular rate. Numerically,

$$K_{q_i} \cong -100 \frac{\text{ft-lb}}{\text{rad}} \tag{24}$$

$$K_{\dot{q}_i} \cong -5 \frac{\text{ft-lb}}{\text{rad}} \tag{25}$$

Figure 5 presents tenative time histories of the joint angles resulting when the above simulation is dropped from an initial altitude of one foot using the above form of control law.

Channel 2 is y, the vertical displacement of the trunk c.g. Released one foot above its equilibrium position, y decreases parabolically under gravity until the toe strikes the ground (see F_{Ty})(Channel 8 shows the heel strike (F_{Hy}) occurring slightly later in time. The vertical heel and toe forces cause y to decelerate and begin to move up (y, Channel 2), the trunk pitches forward (θ, Channel 3), the thigh moves up (γ, Channel 4), the knee bends back (β, Channel 5), the foot angle goes negative (α, Channel 6), and the upper arm pitches forward slightly (δ, Channel 7).

Conclusions

Based on the work we have completed so far, the research program we have outlined still appears feasible; no major alterations appear necessary at the present time.

V. Future Work

1. The simulation results of the preceeding sections are based on equations derived primarily by hand, and therefore are likely to contain a number of analytic and numerical errors. The FORMAC results are now being used to verify and correct these equations.
2. Certain constraints, such as the physiological facts that knees, ankles, and elbows don't bend backwards, have thus far been neglected in the model. They will be included.
3. The control law of equations (23) - (25) will be investigated more thoroughly and modified and refined, in the hope of beginning an investigation of load relief properties in the near future.

References

1. Milhorn, H. T., Jr., "The Application of Control Theory to Physiological Systems", W. B. Saunders Company, Chapter 17: "The Stretch Reflex in Human Muscle Systems", pp. 283-316.

2. Houk, J. C., "A Mathematical Model of the Stretch Reflex in Human Muscle Systems," M. A. Thesis, Massachusetts Institute of Technology, Cambridge, 1963.

3. Fenn, W. O., and Marsh, B. S., "Muscular Force at Different Speeds of Shortening", Journal of Physiology (London), Vol. 85, pp. 277ff, 1935.

4. Bahler, A. S., "Modeling of Mammalian Skeletal Muscle", I.E.E.E. Transactions on Bio-Medical Engineering, Vol. BME-15, No. 4, pp. 249-257, October, 1968.

5. Vickers, W. H., and Sheridan, T. B., "A Dynamic Model of an Agonist-Antagonist Muscle Pair", I.E.E.E. Transactions on Man-Machine Systems, March, 1968.

6. Galiana, H. L., "Modeling the Human Leg in Walking", Master of Engineering Thesis, Department of Electrical Engineering, McGill University, Montreal, Canada, August, 1968.

7. Magdaleno, R. E., and McRuer, D. T., "A Closed-Loop Neuromuscular System Explanation of Force Disturbance Regulation and Tremor Data", Fourth Annual NASA-University of Michigan, Ann Arbor, March 21-23, 1968.

8. DeWhurst, D. J., "Neuromuscular Control System", I.E.E.E. Transactions on Bio-Medical Engineering, Vol. BME-14, No. 3, pp. 167-171, July, 1967.

9. Miery, J. L., "The Vestibular System and Human Dynamic Space Orientation", Doctor of Science Thesis, M.I.T., June, 1965.

10. Young, L. R., "A Control Model of the Vestibular System", I.F.A.C. Symposium on Technical and Biological Problems in Cybernetics, Yerivan, Armenia, U.S.S.R., September, 1968.

11. Gottlieb, G. L., Agarwal, G. C., and Stark, L., "Stretch Receptor Models, I-Single-Efferent Single Affevent Innovation", I.E.E.E. Transactions on Man-Machine Systems, Vol. MMS-10, No. 1, pp. 17-27, March, 1969.

12. McRuer, D. T., and Magdaleno, R. E., "Human Pilot Dynamics with Various Manipulators", Technical Report No. AFFDL-TR-66-138, Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson AFB, Ohio; December, 1966.

13. Wasicko, R. J., McRuer, D. T., and Magdaleno, R. E., "Human Pilot Dynamic Response in Single-loop Systems with Compensatory and Pursuit Displays", Technical Report AFFDL-TR-66-137, Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson AFB, Ohio; December, 1966.

14. Magdaleno, R. E. and McRuer, D. T., "Effects of Manipulator Restraints on Human Operator Performance", Technical Report AFFDL-TR-66-72, Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson AFB, Ohio; December, 1966.

15. McRuer, D. T., Graham, D., Kvendal, E., and Reisener, W., Jr., "Human Pilot Dynamics in Compensatory Systems: Theory, Models, and Experiments with Controlled Element and Forcing Function Variations", Technical Report No. AFFDL-TR-65-15, Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson AFB, Ohio; December, 1966.

16. Shirley, R. S., and Young, L. R., "Motion Cues in Man-Vehicle Control", Fourth Annual NASA-University Conference on Manual Control, University of Michigan, Ann Arbor, March 21-23, 1968.

17. Adams, J. J., "Synthesis of Human Response in Closed-Loop Control Tasks", Fourth Annual NASA-University Conference on Manual Control, University of Michigan, Ann Arbor, March 21-23, 1968.

18. Wier, D. H., and McRuer, D. T., "Models for Steering Control of Motor Vehicles", Fourth Annual NASA-University of Michigan, Ann Arbor, March 21-23, 1968.

19. Crossman, E. R. F. W., "Man-Machine Models for Car Steering", Fourth Annual NASA-University Conference on Manual Control, University of Michigan, Ann Arbor, March 21-23, 1968.

20. Agarwal, G. C., Berman, G., Hogins, M. T., Lohnberg, P., and Stark, L., "Effect of External Loading on Human Motor Reflexes", Fourth Annual NASA-University Conference on Manual Control, University of Michigan, Ann Arbor, March 21-23, 1968.

21. McGhee, R. B., "Some Finite State Aspects of Legged Locomotion", Mathematical Biosciences, Vol. 2, No. 2, pp. 67-84, 1968.

22. McGhee, R. B., "Finite State Control of Quadruple Locomotion", Simulation, Vol. 9, No. 3, pp. 135-140, September, 1967.

23. McGhee, R. B., and Frank, A. A., "On the Stability Properties of Quadruped Creeping Gaits", Mathematical Biosciences, December, 1968.

24. Frank, A. A., and McGhee, R. B., "Some Considerations Relating to the Design of Autopilots for Legged Vehicles", Symposium on Aids to Human Motion, January 21, 22, 1968, sponsored by the General Electric Company, and the U. S. Army Tank and Automotive Command, Warren, Michigan.

25. Tomovic, R., and McGhee, R. B., "A Finite State Approach to the Synthesis of Bioengineering Control Systems", I.E.E.E. Transactions on Human Factors in Electronics, Vol. HFE-7, No. 2, June, 1966, pp. 65-69.

26. Gibson, J. E., di Tada, E. G., Hill, J. C. and Ibrahim, E. S., "Describing Function Inversion: Theory and Computational Techniques", Technical Report TR-EE62-10, Purdue University School of Electrical Engineering, December, 1962.

27. Mosher, Ralph S., "Exploring the Potential of a Quadruped", S.A.E. Paper No. 690191, International Automotive Engineering Conference, Detroit, Michigan, January 13-17, 1969.

28. Blackmer, R. H., Interian, A., and Clodfelter, R. G., "The Role of Space Manipulator Systems for Extravehicular Tasks", Second National Conference on Space Maintenance and Extravehicular Activities, Las Vegas, Nevada, August 6-8, 1968.

29. Whitney, Daniel E., "Resolved Motion Rate Control of Manipulators and Human Prostheses", I.E.E.E. Transactions on Man-Machine Systems, Vol. MMS-10, No. 2, June, 1969, pp. 47-53.